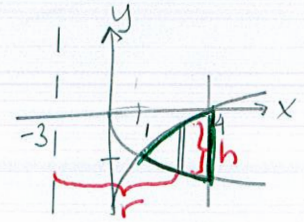
Consider the region bounded by $y = -2 + \sqrt{x}$, $y = -\sqrt{x}$ and $x = 4$. **SCORE: _____ / 17 PTS**

[a] Suppose the region is revolved around the line $x = -3$. Find the volume of the resulting solid.

NOTE: Your final answer must be a number, NOT an integral NOR a sum of integrals.



$$2\pi \int_1^4 (x - -3)(-2+\sqrt{x} - -\sqrt{x})\,dx$$

$$= 2\pi \int_1^4 (x+3)(2x^{\frac{1}{2}} - 2)\,dx$$

$$= 2\pi \int_1^4 (2x^{\frac{3}{2}} - 2x + 6x^{\frac{1}{2}} - 6)\,dx$$

$$= 2\pi \left(\tfrac{4}{5}x^{\frac{5}{2}} - x^2 + 4x^{\frac{3}{2}} - 6x\right)\Big|_1^4$$

$$= 2\pi \left(\tfrac{4}{5}(32-1) - (16-1) + 4(8-1) - 6(4-1)\right)$$

$$= 2\pi \left(\tfrac{124}{5} - 15 + 28 - 18\right)$$

$$= 2\pi \left(\tfrac{124}{5} - 5\right)$$

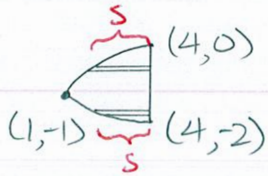
$$= 2\pi \left(\tfrac{99}{5}\right)$$

$$= \boxed{\frac{198\pi}{5}}$$

$$-2+\sqrt{x} = -\sqrt{x}$$
$$2\sqrt{x} = 2$$
$$\sqrt{x} = 1$$
$$x = 1$$

[b] Suppose the region is the base of a solid. Cross sections perpendicular to the $y-$axis are equilateral triangles. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\frac{\sqrt{3}}{4}\int_{-2}^{-1} (4-y^2)^2\,dy + \frac{\sqrt{3}}{4}\int_{-1}^{0} (4-(y+2)^2)^2\,dy$$



$y = -2+\sqrt{x}$ $\quad$ $y = -\sqrt{x}$

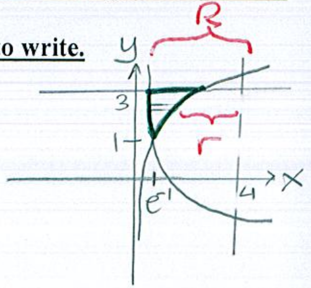
$y+2 = \sqrt{x}$ $\quad$ $x = y^2$

$x = (y+2)^2$

The region bounded by $y=-\ln x$, $y=2+\ln x$ and $y=3$ is revolved around the line $x=4$. SCORE: _____ / 6 PTS

Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the resulting solid **using as few integrals as possible**.

NOTE: Do NOT use geometry shortcuts (ie. volume formulae) to reduce the number of integrals you need to write.



$y=-\ln x$ $\quad$ $y=2+\ln x$

$-y=\ln x$ $\quad$ $y-2=\ln x$

$x=e^{-y}$ (½) $\quad$ $x=e^{y-2}$ (½)

(1) $\pi\int_1^3 \left((4-e^{-y})^2-(4-e^{y-2})^2\right)dy$ (½) (½) (½) (½)

(1½) BOTH RADII SQUARED & CORRECT ORDER OF SUBTRACTION

$-\ln x=2+\ln x$

$-2=2\ln x$

(½) $\ln x=-1$

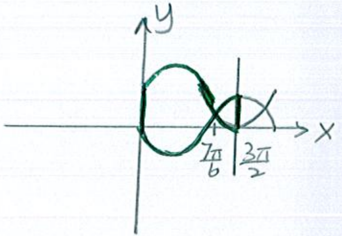
$x=e^{-1}$

$y=-\ln e^{-1}$

$=1$

Find the area between the curves $y = 1 + \sin x$ and $y = -\sin x$ over the interval $[0, \frac{3\pi}{2}]$. **SCORE: _____ / 7 PTS**

NOTE: Your final answer must be a number, NOT an integral NOR a sum of integrals.



$$\int_0^{\frac{7\pi}{6}} (1 + \sin x - {}^{-}\sin x)\,dx + \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} (-\sin x - (1 + \sin x))\,dx$$

(1) (1)

$$= \int_0^{\frac{7\pi}{6}} (1 + 2\sin x)\,dx + \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} -(1 + 2\sin x)\,dx$$

(1) (1)

$$= (x - 2\cos x)\Big|_0^{\frac{7\pi}{6}} + {}^{-}(x - 2\cos x)\Big|_{\frac{7\pi}{6}}^{\frac{3\pi}{2}}$$

($\frac{1}{2}$) ($\frac{1}{2}$)

$$= \left(\frac{7\pi}{6} - 2 \cdot {}^{-}\frac{\sqrt{3}}{2}\right) - (0 - 2 \cdot 1) + \left(-\frac{3\pi}{2} + 2 \cdot 0\right) - \left(-\frac{7\pi}{6} + 2 \cdot {}^{-}\frac{\sqrt{3}}{2}\right)$$

$$= \frac{7\pi}{6} + \sqrt{3} + 2 - \frac{3\pi}{2} + \frac{7\pi}{6} + \sqrt{3}$$

($\frac{1}{2}$)

$$= \frac{5\pi}{6} + 2\sqrt{3} + 2$$

($\frac{1}{2}$)

$1 + \sin x = -\sin x$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$ (1)

$x = \frac{7\pi}{6}$